Towards a teaching sequence for mental computation strategies

Ann and Johnny Baker

Natural Maths

It is some 10 years now since we first started to investigate why it was that teachers teach mental computation strategies but students don't use them when they are working on their own. During one lesson I watched a teacher with children on the carpet labour over the *count on 1, 2 and* 3 strategies and then send the students away to complete a text book page with 'sums'. As I looked around the class what did I see? I saw students putting up their fingers and counting all, not counting on.

It was this particular event that led me to invent and trial the 'secret code'. For every strategy that we teach we also teach the secret code for it, **co** for *count on*, **d** for *double* and so on. Details of the secret code can be found in Mental Computation (Lower) and Natural Maths Strategies for Parents: Book 1. The students really like using the secret code and it gives a clear message that we are expecting students to actually look for an effective strategy and communicate what that strategy is. This on its own is having profound effects on a student's mental computation. It also allows the busy teacher to look at a student's work and see the secret code written there. Diagnostic information about which strategies a particular student is using can be made and early interventions or new challenges can be planned for. After the initial success of introducing the secret code and taking a rigorous approach to teaching and applying mental computation strategies we enlisted the support of some of our friendly teachers to see just what students are able to learn and do in the early years.

For the last three years we have been working with teachers across several large schools to find out exactly what it is reasonable to expect young students to be able to do with number by the end of each early year at school. We have focused on strategies, their language and meta-language to develop a developmental sequence that guides what we are introducing when in terms of addition and subtraction. We have not been too focused on formal recording preferring to place the focus on number sense and fluency. The following outline describes what we have introduced when, in what order and what the results have shown.

Prep

While a first encounter with maths should be informal, we find that almost all prep students are ready for:

Subtising – using dice, pictures and children's own art work to encourage the ability to look and suddenly know how many in a group without having to count. This is the foundation of all later number sense and should be fully developed during prep. Initially we use floor dice, throw them and drop a magic cloth immediately over the top. Students then draw what they saw. We find we can move on to two dice quite quickly. Notice that there is insufficient time to count the dots – we do not at this time want to encourage count all strategies as these are persistent and lead to finger counting.

Counting All – when there are too many things to be able to subitise or they are poorly arranged and have to be counted then often count all is the only strategy that works. During this time one to one correspondence is important and can be observed.

Count On – when subitising is developed it is quite easy to subitise one group or part of a group of objects and then count on. And of course this is far more efficient that counting all and possibly more reliable. This requires that the students can break the counting sequence. Teachers can model the 'breaking the counting sequence' in everyday situations, for instance "6 people are ready on the carpet, let's count on as the others arrive, 7, 8, 9 ..."

Doubles – when playing dice games the idea of doubles can be introduced quite easily.

Many prep students find it very engaging to spot doubles.

Summary – when Prep students are engaged in daily activities related to number and use numbers in their play all but 3 or 4 in any group manage to become fluent with the strategies listed above and to be able to use the meta-language to explain what they did, for example:

"I subitised 4 and then counted on 2 more, 5, 6."

Between 4 and 8 students in any prep class also move beyond this, inventing strategies such as near doubles and skip counting (if they have been immersed in counting in 2s or 5s) and will be using numbers well past 20.

One of the major advantages of this approach is that we are seeing the majority of students arriving in Year One the following year with a deep understanding of the concept of altogether and as such are ready to begin more formal addition situations. Students who have not had this approach often have no idea of altogether, so when they see two groups one with 4 and one with 2 in them they are likely to name each group and not to combine them and indeed look confused when asked how many altogether?

Strategic Maths Number: Beginning Level has explicit teaching segments with Interactive Whiteboard support to introduce early addition situations to Prep students.

Year One

Assuming that a firm base has been laid in prep the above strategies can be maintained and further developed.

Count on 1, 2, 3 – early in Year One students should be discouraged from using count all or from counting on more than 1, 2 or 3, because other strategies now come into focus as described below.

Turnarounds – when playing with dice it is possible for a 2 and a 6 to be thrown. It is inefficient to start at 2 and count on 6, so we need to introduce the idea of counting on from the largest number, **6**, 7, 8. Playing with dice is good for this because the dice can be physically moved before the addition is carried out.

Doubles are consolidated and extended beyond double six to match games played with ten sided dice or nines dominoes.

Rainbow facts – the visual image of the rainbow is used to develop fluency with number pairs that make to ten

Near doubles – again the dice are useful here because showing the near doubles in dice format makes a visual representation that all students can see and get.

Friendly Numbers – 10 is a friendly number (as is any number ending in zero) because it is easy to add onto and the pattern of replacing the zero with the number being added can be easily spotted and applied.

Summary – when we rigorously introduce and expect students to use the secret code whenever they carry out an addition we are giving some very clear signals to them. These expectations seem to make a big difference in the extent to which students apply mental computation strategies and apply then to problem situations. In effect we are saying to students 'look for and apply the most efficient strategy that **you can** to the situation'. Because students use the secret code to identify the strategy that they use, even when we are not able to talk to a student during the lesson we can see from their work after the lesson what strategies they did use. This allows for early interventions, helping a student move on from always counting on to using another strategy where appropriate.

We have found that all but 3 or 4 students will be fluently using, count-ons, turnarounds, doubles, and rainbow facts. Some (roundabout 6 to 10) students will be fluently using the full range described above.

Strategic Maths Number Lower Primary Book 1 has explicit teaching strategies supported by Interactive Whiteboard activities to introduce mental computations strategies to Year One students.

Year 2

Assuming that a firm base has been developed in Year One we find that Year Two students further consolidate and apply the strategies described above and develop automaticity with them. That is they no longer need prompts they just know the answers instantly. They will also add the following to their repertoire of strategies:

> **Bridge through 10 –** if the number fact 8 + 5 is not recalled quickly, we can break the addition into two parts. First, split the 5 into 2 + 3 and that allows us to use the rainbow fact 8 + 2 followed by adding on to a friendly number 10 + 3 to get the answer 13. Often, students will invent the bridge through ten strategies by themselves, but even so, early in Year 2 is a good time to formalize it with all students and give it its proper name.

Extended number facts – if a firm foundation has been laid students will begin to apply these strategies now to extended number facts, this means that for 40 + 20 they will know to break the tens counting sequence and count on in tens, For 30 + 30 they will know that double 3 is 6 so double

> **Count on 10, 20, 30 –** early in Year Two students should be encouraged to extend the count-on strategy to counting on in 10s.
>
> **Doubles and Near Doubles –** should also be extended to cover the 10s, for example 4 + 4 = 8 should be extended to 40 + 40 = 80.
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> **Rainbow facts –** can also be extended to the 10s making them rainbow facts to 100.
>
> **Friendly Numbers –** any number that ends in zero can be called a friendly number because it is easy to add onto and the pattern of replacing the zero with the number being added extends naturally to larger numbers.

The strategies can also be extended to subtraction and the associated language for that should also be used. For example, to find 7 – 2, we can use a **count back** strategy 7, 6, 5. Doubles become halves, so that 16 – 8 is thought of as a halving process. And finally, the rainbow facts are also used in reverse so that 10 – 4 is seen as using the rainbow fact 6 + 4 = 10.

Strategic Maths Number Lower Primary Book 2 has explicit teaching strategies supported by Interactive Whiteboard activities to introduce mental computations strategies to Year 2 students.

Year 3

The mental computation strategies that enable us to work with 2-digit numbers can be introduced in Year 3. While doing this, it is important to use **value-based** rather than **digit-based** language. What does *value-based language* mean? It is best explained by an example. To add 35 + 47, we need to refer to the 3 and the 4 as thirty and forty, and we might speak this addition out loud as:

> "Thirty plus forty is a near double and makes seventy. I can use a bridge through 10 to find five plus seven. It is five plus five plus two, making twelve. So in all I have seventy plus twelve which makes eighty two."

If the same addition was carried out as a vertical algorithm, it would be described in digit-based language as follows:

> "Five plus seven makes twelve. Put down the two and carry the one. One plus three plus four makes eight. So the answer is eighty two."

The new strategies at this stage are:

> **Landmark numbers –** the numbers 25, 50, 75 and 100 are called landmark numbers and knowing how landmark numbers combine is a useful strategy when adding 2-digit numbers. For example, 26 + 52 can be thought of as **25** + 1 + **50** + 2 = **75** + 3 = 78.
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> **Tallies –** While tallies offer an effective way of gathering statistical data, they can also be used to great effect when demonstrating the use of repeated addition for multiplication. Pages 24 and 25 in the Parent book give examples of this.
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> **Rainbow facts –** when adding a list of numbers, you can encourage the students to look for rainbow facts that will make the additions simpler and less error-prone. For example, in 83 + 24 + 47, spotting that 80 and 20 are a rainbow fact, makes is easy to add the 10s to give 140. Similarly, the 3 and the 7 are a rainbow fact, so the 1s add to 14.
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> **Number splitting –** the example given for landmark numbers above is a form of number splitting that reduces the addition to an addition that we can feel comfortable with. Similar types of number splitting can be used when we can see, for example, that a rainbow pair could be made. For example to add 43 + 38, we might see that thinking of this as 43 + 37 + 1 gives us a rainbow fact (3 + 7) that we can deal with first and move on fr4om there to get the answer 81.

All of the strategies mentioned so far are described in Natural Maths Strategies for Parents: Book 1. Strategy lessons are also suggested in the Strategic Maths: Number series.

Year 4

The strategies used in Year Three for working with 2-digit numbers need to be consolidated in Year 4. We also need to be making links between addition and other aspects of calculation. For example, the landmark numbers 25, 50, 75 and 100 can be linked with decimal fractions 0 25, 0·5. 0·75 and 1 and with the percentages 25%, 50%, 75% and 100%. This association can also be made with time, where the landmarks as 15, 30 and 45.

There are also some new strategies that the Year Four student is ready for. These include:

Rounding – rounding up or down to the nearest friendly number is an important skill, as it forms the basis of estimation, a skill that enables student to check the reasonableness of an answer.

Round and Adjust – frequently, a mental computation is made easier if one of the numbers is rounded to a friendly or landmark number before the addition is carried out. For example, to find 29 + 57, we can round the 29 to 30, add 30 to 57 giving 87 and then adjust by subtracting 1 to give 86, because we know that we have added 1 to much. Round and adjust can also be thought of as **round and compensate** if to find 29 + 57, we round the 29 to 30 and compensate by reducing the 57 by 1 giving 30 + 56, which is a much easier mental computation.

Flexibility with Representations

As well as having a range of computation strategies, students should also be able to represent their computations in a number of different ways. The representations include chunking, empty number lines and the use of grids and all are important in giving the students flexibility when they are faced with a problem.

In Conclusion

So, why do we believe mental computation strategies to be so important? Our preparedness to engage with a problem hinges on the thinking strategies that we have at our disposal and which we can bring to bear on the situation. If the range of strategies is very limited, then we are put off making that first step of engagement. By equipping students with a versatile range of mental computation strategies, we are making it possible for them to engage with the task at hand, especially if it is unfamiliar to them.

There is another important feature of the mental computation strategies that we have discussed above. It is that they are very robust. Once acquired, the strategies continue to give good service long after their first use. Indeed, these are the strategies that we, as adults, often use when faced with the need to make a calculation. Unlike some of the devices that we use to introduce arithmetic skills and understanding (e.g. bundling), devices that are abandoned once we move on, mental strategies have a very long shelf life.

